Lecture 28

Tuesday, November 15, 2016 8:57 AM

<u>Recall</u>

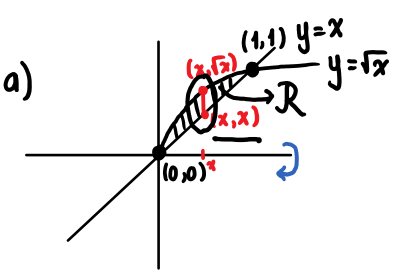
<u>DEFN</u> Let S be a solid that lies between $x=a$ and $x=b$. If the cross-sectional area of the plane P_x, through x and perpendicular to the x-axis, is $A(x)$, then the volume V of S is

$$V = \lim_{n\to\infty} \sum_{i=1}^{n} A(x_i^*)\,\Delta x = \int_a^b A(x)\,dx$$

provided $A(x)$ is integrable.

<u>Ex</u> Let $\mathcal{R}$ be the region bounded by the curves $y=\sqrt{x}$ and $y=x$. Find the volume of the solid obtained by rotating the region about a) the x-axis b) y-axis

a)



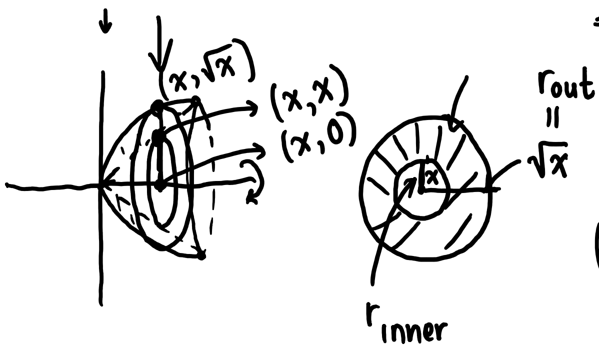
First points of intersection

$\sqrt{x} = x \Rightarrow x = x^2$

$\Rightarrow x - x^2 = 0 \Rightarrow x(1-x) = 0$

$\Rightarrow x = 0$ or $1 - x = 0$

$\Rightarrow x = 1$



(Washer)

<u>Q</u> $A(x) = ?$

$A(x)$ = Area of bigger disc − Area of smaller disc

disc

$$= \pi r_{outer}^2 - \pi r_{inner}^2$$

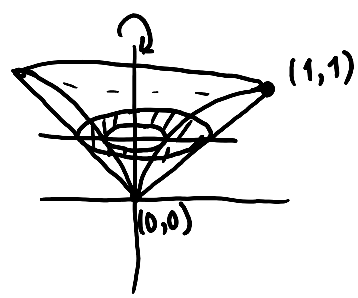
$$= \pi(\sqrt{x})^2 - \pi(x)^2$$

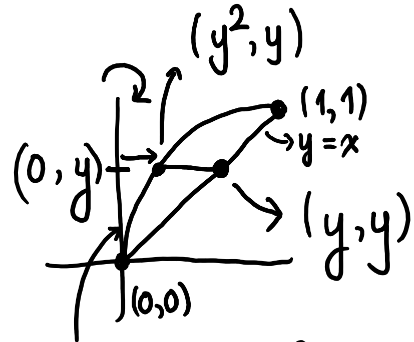
$$= \pi(x - x^2)$$

$$V = \int_0^1 \pi(x - x^2)\,dx = \pi \int_0^1 x - x^2\,dx \ldots$$

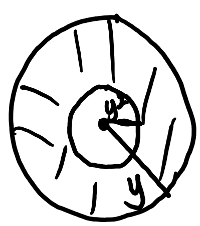
$$= \frac{\pi}{6}.$$

b)





$$y = \sqrt{x} \Rightarrow x = y^2$$



$$r_{inn} = y^2$$
$$r_{out} = y$$

$$A(y) = \pi\left(r_{out}^2 - r_{inn}^2\right) = \pi(y^2 - y^4)$$

$$V = \int_0^1 \pi(y^2 - y^4)\,dy = \pi \int_0^1 y^2 - y^4\,dy$$

$$= \frac{2\pi}{15}$$

6.4: Work

Technical defn of work depends on force.

In general, if an object moves along a straight line w/ position func $s(t)$, the the force F on the object (in the same direction) is given by

$$F = \overset{\text{mass}}{m} \, \frac{d^2 s}{dt^2} \; (\text{acc}^n)$$

(Newton's Second Law of motion).

$m = kg$, $s(t) = m$, $t = s$,

unit of $F = Kgm/s^2 = N$.

If acceleration is constant, force is constant.

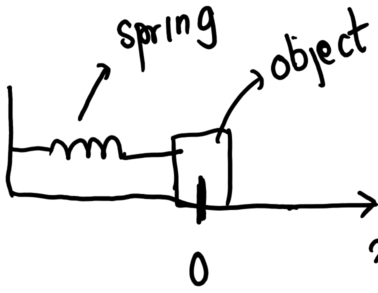
$W = F \cdot d$ → distance. $= N \cdot m = J$ (Joule)

RmK If F = pounts (lbs), $d = ft$, $W = ft\text{-}lbs$

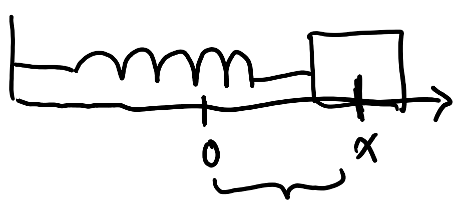
DEF Suppose that object moves along the x-axis in the positive dirn from $x = a$ to $x = b$ and at each pt x in $[a, b]$, a force $f(x)$ acts on the object. The work done in moving the object from a to b is given by the object from a to b is given by

$$W = \lim_{n\to\infty} \sum_{i=1}^{n} f(x_i^*)\Delta x = \int_a^b f(x)\,dx .$$

Hooke's Law



Natural posn



Stretched String

Force required to maintain a spring stretched x units beyond it's natural length is proportion to x:

$$f(x) \propto x \Rightarrow \underline{f(x) = Kx} .$$

Ex A force of 40N is required to hold a spring that has been stretched from it's natural posn of length 10 cm $\rightarrow$ 0.1m to a length of 15 cm $\rightarrow$ 0.15m.

How much work is done in stretching the spring from $\underbrace{15\text{ cm}}_{0.15\text{m}}$ to $\underbrace{20\text{ cm}}_{0.2\text{m}}$?

Soln $f(x) = kx$

Q tells us when $x = 0.05\text{ m}$, $f(x) = 40\text{ N}$

$\Rightarrow 40 = k.(0.05) \Rightarrow k = 800$

$\Rightarrow f(x) = 800x$

amount stretched (in m)

$$W = \int_{0.05}^{0.1} 800x \, dx$$

$$= 800 \int_{0.05}^{0.1} x \, dx = 3\text{J}$$